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JOSEPH C. CAIN
SHIRLEY J. HENDRICKS
ROBERT A. LANGEL

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A PROPOSED MODEL FOR THE INTERNATIONAL GEOMAGNETIC REFERENCE FIELD - 1965*

by

Joseph C. Cain

Shirley J. Hendricks

Robert A. Langel

William V. Hudson

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ABSTRACT

A best current model of the main geomagnetic field is presented as a response to a need for an "International Geomagnetic Reference Field". This model is described by a series of 120 spherical harmonic coefficients and their first and second time derivatives from an epoch 1960.0. It was derived from a sample of all magnetic survey data available from the interval 1900-1964 plus a recent global distribution of preliminary total field observations from the OGO-2 (1965-81A) spacecraft for epoch 1965.8. A duplicate data selection was made and the resulting field model compared with the first to help evaluate the minimum error. It was noted that the root-mean-square difference between the two models was about 30γ in the force components, 0.04 degrees in dip and 0.3 degrees in declination at the earth's surface for 1965.0.

I. Introduction

There have been increasing requests by scientists working in such diverse fields as crustal geology and magnetospheric particle physics for one "standard" reference geomagnetic field description. The crustal geophysicist needs a field description adequate to reference his measurements of surface field, usually absolute force, so that the background or "main" field can be simply evaluated and subtracted from his data. Some studies are aimed at investigating only the very local crustal structure whereas others wish to have a reference sufficiently smooth that longer scale "regional" anomalies can be seen. The accuracy requirements are generally not great and can be easily satisfied if the reference is accurate to 100γ.

The particle physicist, however, is only trivially interested in the surface field and instead wishes to know the total ambient vector field to the limits of the magnetosphere as accurately as it can be predicted (Cain, 1966).

Both these and other users note that the salient features of a reference field are that it not be altered too frequently and that it be the product of international agreement.

At a colloquium on the World Magnetic Survey held in October, 1966 at Herstmonceux, England, B. R. Leaton proposed that considerations for an "International Geomagnetic Reference Field (IGRF)" be culminated for adoption at the 14th General Assembly of the International Union of Geodesy and Geophysics to be held in 1967. To begin implementation of this proposal Dr. Leaton has distributed to various committee members of the Working Group on the Analysis of the Geomagnetic Field (Chairman, Dr. A. Zmuda, under Commission III of the International Association of Geomagnetism and Aeronomy) and other interested scientists a comparison of several recently published field models and a first weighted average approximation for the epoch 1965.0. Since this distribution, there has been lively correspondence representing different views on the solution to the problem. Since the ways of evaluating the field, accuracy, and other requirements vary widely according to the user, our proposal to meet the many requests is to see whether agreement can be reached on a field model which most accurately represents the available data. Once this model is agreed upon, it is a relatively straightforward matter to cast the results in a form suitable to the user.

To be inclusive of the most demanding accuracy requirements it is necessary to take into account some measure of secular change. While some users will be quite content to use one simple model over even a decade, others will find that such changes as 5 to 10γ ($1\gamma = 10^{-5}$ Gauss = 1 nanoweber/m^2) per month in total field at the surface must be accounted for in their application. Also, since the earth's radius is some 20 km. greater at the equator than at the poles, accurate representations cannot depend on approximating the earth as a sphere (Kahle, Kern, and Vestine, 1964, 1966).

Fortunately, the program that we have been pursuing over the past few years has required us to attempt to determine reference field models whose absolute error is no more than a few gammas at low satellite altitudes. We need such accuracy to allow us to use field measurements by satellites to make studies of the temporal perturbations in the field from sources external to the earth and also to study large scale crustal features and short period changes in the core field itself.

Although we have not yet achieved a model that is as accurate as that required, steady progress has been made in this direction and we believe that the model described herein should be sufficient for most purposes other than our own. It is thus presented as an answer to a request for the most accurate available model known to us even though the errors in future determinations may very well be up to an order of magnitude less. As will be seen subsequently, the reason for a higher confidence at this time is the inclusion of comprehensive, albeit preliminary, data from the OGO-2 satellite, and the inclusion of enough terms to greatly improve the estimates of secular change over those in previous models.

II. Review of Past Work

The comparisons that have previously been made between the survey data taken for recent epochs and some of the recent field models have shown a steady improvement with an increasing number of coefficients. Using the set of magnetic survey data available over the interval 1945-1965, Cain (1966) obtained the following average rms deviations between the data and available models:

Field Model	rms (γ)
J & C (Jensen and Cain, 1962)	440
GSFC(4/64) (Cain, et al., 1965)	270
LME (Leaton, Malin, and Evans, 1965)	260
GSFC(7/65) (Cain, 1966)	220

Since then, the field GSFC(9/65) was produced (Hendricks and Cain, 1966) as an equivalent to GSFC(7/65) having about the same agreement with the survey data but without the possibly fictitious external terms.

A previous comparison made by Cain et al. (1965) had shown that the match to recent data by fields other than LME or the GSFC(4/64) were clearly worse and do not warrant further consideration.

There has recently been published a USC&GS model (Hurwitz et al., 1966) which was used as the basis for the 1965 United States World Magnetic Charts as published by the U.S. Naval Oceanographic Office. Although this model could well be a good fit to the field components at the earth's surface for 1965.0, it cannot be considered as suitable as an IGRF since there is no provision for evaluation at other epochs, it assumes the spherical earth approximation, and, as noted by the authors, it does not extrapolate well to low satellite altitudes.

In considering our recommendation for an IGRF from published material we would then suggest the adoption of the GSFC(9/65) model were it not for its deficiencies, some of which have been remedied in the present work. The main problem of this older field is that although the number of coefficients was raised to 99 (expansions of potential function of degree n and order m of nine), computer speed and core limitations then only allowed determination of the linear secular change of the first 48. The higher order coefficients were thus only the mean values over the period 1945-1964 comprising the data set. Also, the data for this short period were so spotty that it was unlikely that the secular change estimates were more than gross approximations. It was not then possible to extend the range of the data used in the fit to obtain a better secular change estimate since the number of parameters would have had to be further expanded to include parabolic terms in time.

A major relief to these computer limitations presented itself when we obtained access to a new computer (Control Data Corporation Model 6600) which appeared to have the core size and speed to attempt a fit to a sufficiently long set of survey data with enough parameters to adequately represent the field including its secular change.

III. Method of Analysis

The bare outline of the weighted least squares technique used to fit the survey data is given by Cain et al. (1965). This outline plus a description of some of the detailed equations appears in the appendix of this paper.

The technique used has the result of minimizing the sum

$$\chi^2 = \sum_{\text{data}} (C_o - C_c)^2 w_i$$

where the C_o is an observed component of the field, C_c is the computed value, and w_i a weighting factor. The observed components are Z, H, F, D, and I where the first three force components are measured in gammas and the last two in degrees. In order to make the D and I measures commensurate with the force components, the $(C_o - C_c)$ values were multiplied by computed values of H and F respectively. The quantities minimized were thus ΔZ, ΔH, ΔF, $H\Delta D$, and $F\Delta I$, each weighted by w_i.

Statistical theory tells us that in order to obtain the minimum value of χ^2, the w_i should be inversely proportional to the average of the $(C_o - C_c)^2$. In the laboratory experiment this normally implies that the minimum variance estimates are obtained by weighting inversely as the square of the average error. In such experiments it is common to discard disturbed data so that only the accuracy of the measurement itself need be considered. In the paper by Cain et al. (1965), the weights applied were chosen on the basis of the estimated accuracies of measurement. We were then weighting by $w_i = 1/\sigma$ (where σ was the estimated accuracies) instead of the more statistically correct $1/\sigma^2$ weighting. When this oversight was pointed out to us by F.J. Lowes (private communication, 1965) we were reluctant to change since we had such a small error assigned to magnetic observatory data that using the "correct" weighting method would have cinched the fit very tightly to these observations at the expense of the others. The problem lay in the fact that although the near earth surface data is highly disturbed by crustal anomalies, the fitting function only attempts to fit the smooth background field. Thus the anomaly "noise" constitutes a permanent disturbance to the near surface data apparently only becoming insignificant a few tens of kilometers above the earth's surface (Davis et al., 1965). Even though the magnetic observatories are located at sites which may have a smooth horizontal gradient in that local area and the measurements are normally taken with great care and accuracy, there is no guarantee that the area is representative of the average field over a larger area.

Unless some spatial smoothing of the data is carried out to help eliminate the anomaly noise, it is clear that the minimum of χ^2 can be achieved only by weighting inversely as the square of the rms scatter of the data. To estimate this scatter we made some test fits on samples of the data and estimated that there were no appreciable differences between the surface (including observatory) and aircraft data residuals. The F and H observations always gave about 200γ whereas Z appeared to be systematically higher. We thus decided to base the weights on the approximate scatter of data by class in the data sample used. The actual table of weights used will be discussed subsequently along with the selection of data.

Error Estimates

In the paper by Cain et al. (1965) an attempt was made to estimate the standard error of each of the coefficients g_n^m, h_n^m, $\dot{g}_n^m$, and $\dot{h}_n^m$ by using statistical estimates of the internal consistency of the data assuming a gaussian error distribution. The precise way in which this is done appears to depend on the way that one weights the data. In order to test the validity of our error estimates we initially planned to make determinations based on several independent selections of the data and compare the resulting coefficients and computed fields. As will be seen subsequently, the data are not sufficiently abundant and well distributed to obtain even two adequate separate and distinct samples.

However, in order to obtain at least a minimun estimate of the differences that could arise by selecting the data in different ways, it was decided to create two equivalent data samples with as little overlap of observations as possible.

IV. Computations

Data Base

The basic magnetic survey data used included the total set contained in World Data Center A for Geomagnetism and provided to us on digital magnetic tape by the United States Coast and Geodetic Survey's Geomagnetism Division. The distribution of these data through 1962 is given by Cain and Hendricks (1964). These data include all project MAGNET observations through 1963 (USNOO, 1965) and a scattering of observations for 1964. High altitude data in the data center at the time this tape was received include the Vanguard 3 observations, Alouette gyrofrequency measurements over Canada, and a few scattered sounding rocket results. In the course of performing test fits on random samples of these data the rms scatter was investigated and the following approximate results obtained when data with ΔC (= ΔF, ΔZ, ΔH, $H\Delta D$, or $F\Delta I$) > 2000γ were rejected:

Table 1

Component	σ (rms)	
D	1°	
I	0.3°	
H	200γ	
Z	280γ	
F	50γ	Vanguard 3
	200γ	All other

The data were then edited by deleting observations where ΔC exceeded 1000γ, a figure of about 5σ. This selection was monitored to be sure that no data were deleted that might be representative of an area. A detailed listing showed that only about 1% of the data were rejected and the observations all appeared to be isolated and due either to recording error or crustal anomalies.

In addition to the basic data set received from the USC&GS, a set of preliminary data from the OGO-2 satellite was obtained (Cain et al., 1967). These data covered the period October 29 - November 15, 1965 which was very quiet magnetically. The observations were considered preliminary in that the final time corrections were not applied and the orbital positions used may have errors of the order of a kilometer.

These satellite data were fit with an increasing number of spherical harmonics with the result that the residuals in ΔF decreased with an increasing number of coefficients to 7γ at $n = m = 10$. A further increase from these 120 coefficients to 143 ($n = m = 11$) showed less than a 10% improvement. The selection of OGO-2 data for addition to the master data set was made using a 99 coefficient test set which gave a 10γ rms residual in ΔF. Data were added to the previously edited data tape using each 10th observation (5 sec or about 35 km. interval for the orbit which ranged in altitude from 410 to 1510 km.) if $\Delta F < 30\gamma$ from this fit. This selection omitted only a very few data which were judged on comparison with surrounding observations to be erroneous.

Data Selection

This first data sample was obtained from the master data set by dividing the earth into a grid of areas spaced 2^{o} in longitude and varying latitude so that each lune of longitude was divided into 200 equal areas. These blocks start at 0 to 0.57^{o} in latitude on both sides of the equator, become nearly square at 60^{o} latitude, and the last sections extend from 82^{o} to the poles. A selection was then made of the first observation per year which fell in each of these area blocks. The resulting data set No. 1 contained approximately 77,400 observations of one or more components each, the positions of which are plotted separately in Figure 1 for the epochs 1900-1930 (Figure 1a), 1930-1950 (Figure 1b), and 1950-1965 (Figure 1c). These figures show graphically the important data gaps for each period. For the early periods (Figure 1a) the only large areas of neglect were both polar regions, northern Africa and Asia Minor. The intermediate two decades (1930-1950) acutely show the loss of the survey Ship Carnegie which was the main supplier of ocean data for the previous decades. Except for scattered observations, the high latitudes continue to remain barren of data. Of course, this map projection gives greater enlargement to the polar regions but the areas missed are still considerable when viewed on a globe.

The last figure shows an almost gapless data distribution. However, it is easy to see beneath the north-south tracks of the OGO-2 satellite and note that up to the 1965.8 epoch of these observations, the distribution contained some large gaps.

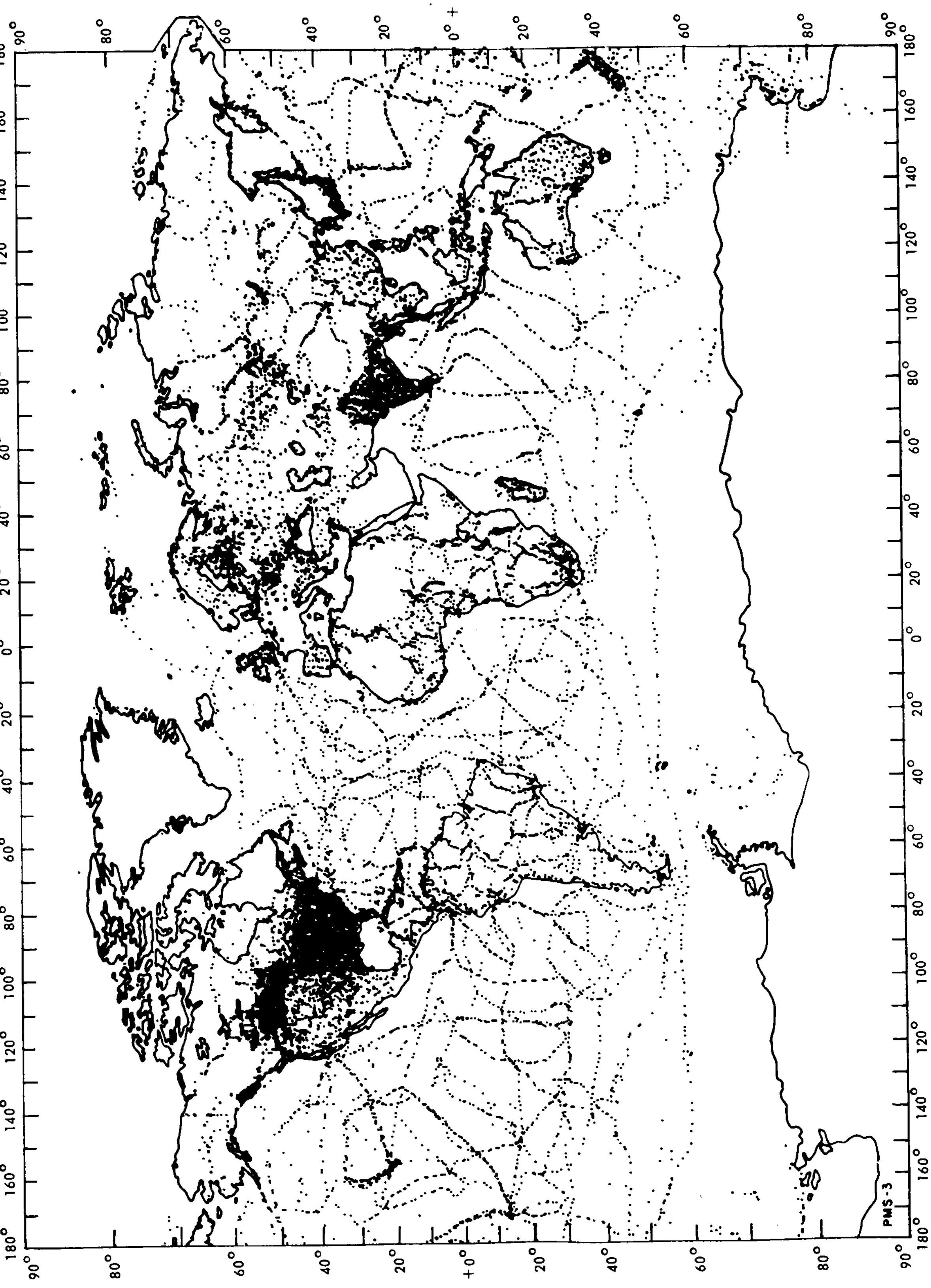

Figure 1a. Data Set No. 1 (1900-1930)

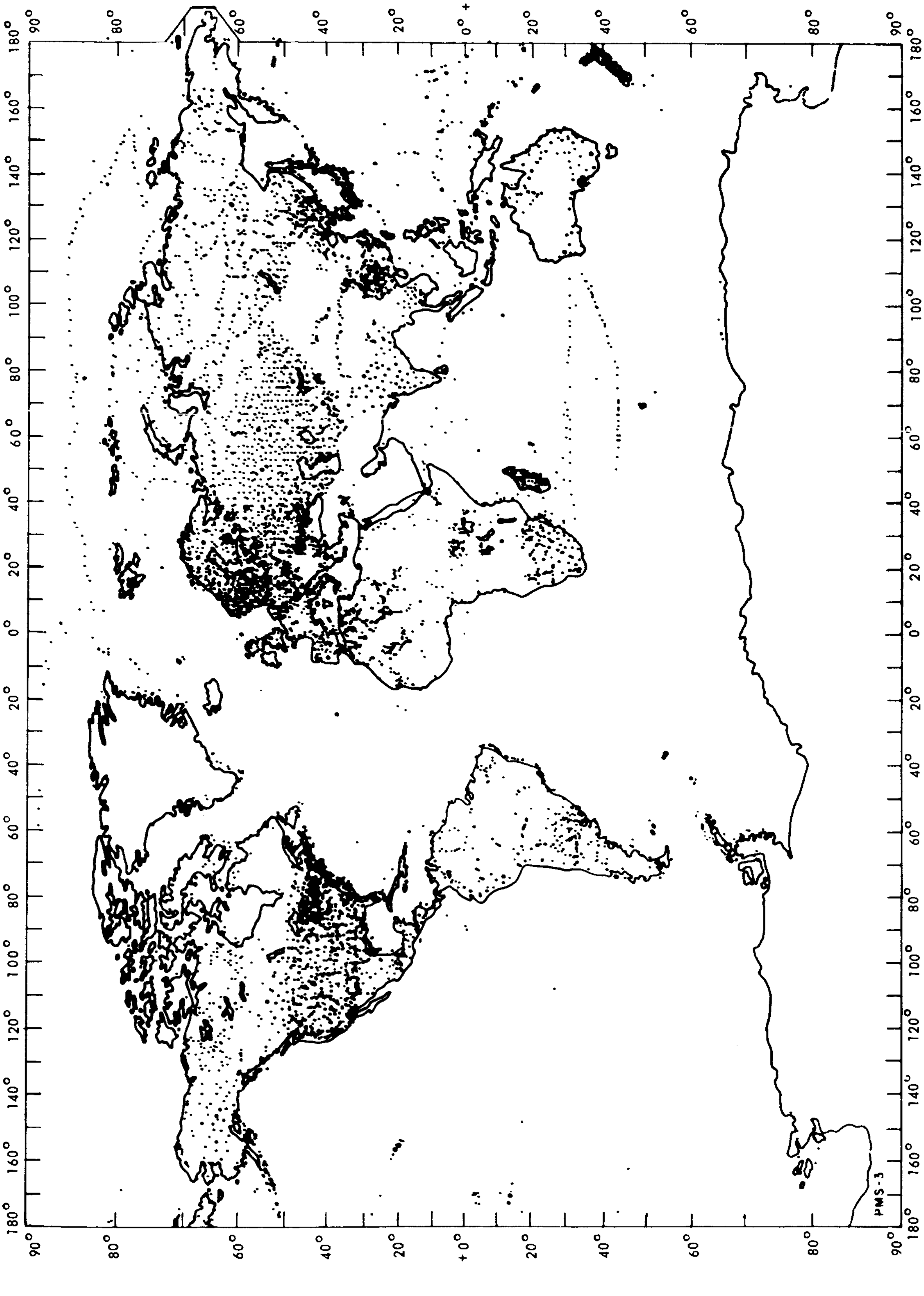

Figure 1b. Data Set No. 1 (1930-1950)

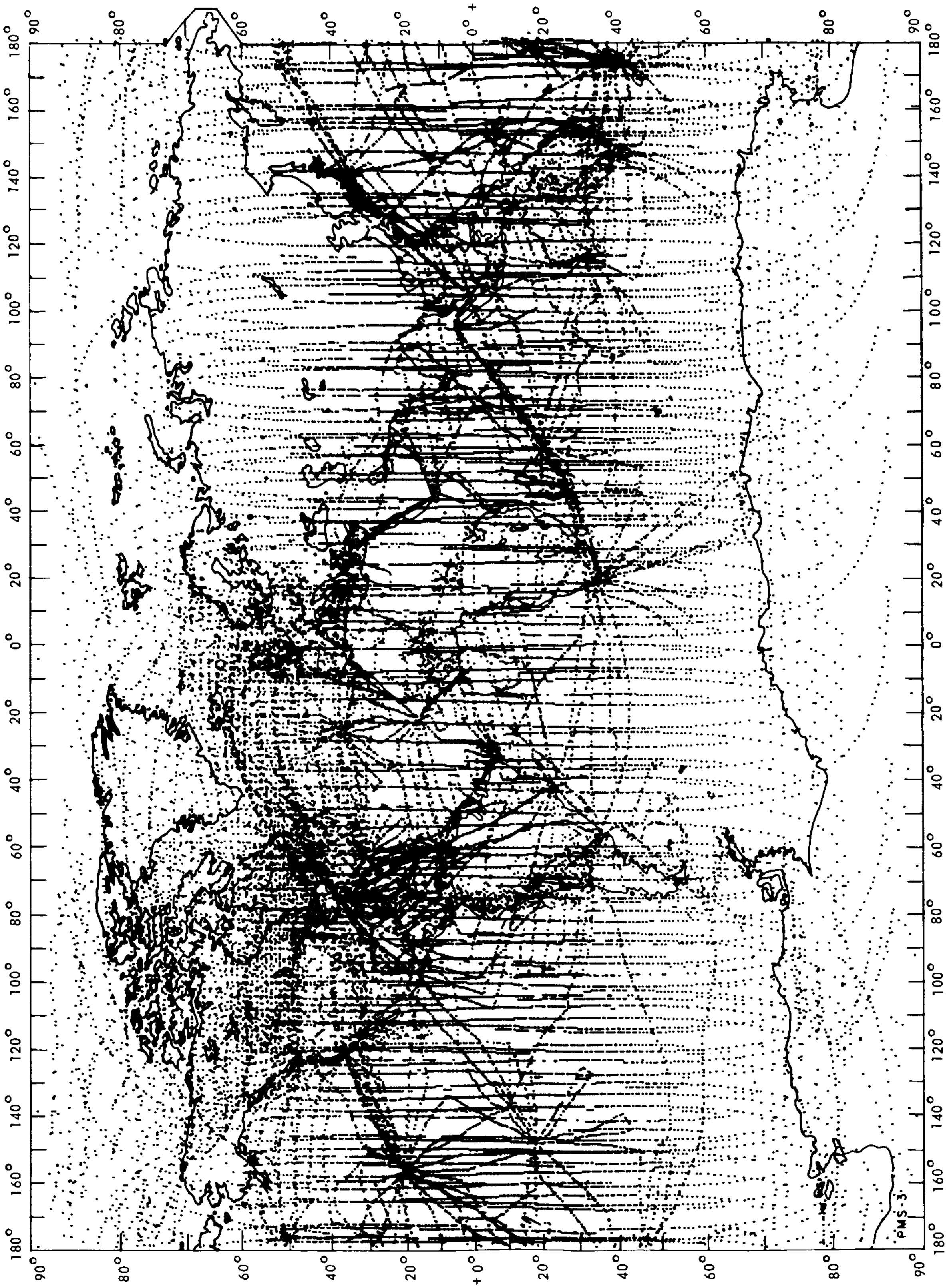

Figure 1c. Data Set No. 1 (1950-1965)

The distribution of observed components by decade is given in Table 2. The data for 1965 consists of 22322 OGO-2 observations and 12 observatory annual means.

Table 2

Interval	D	I	H	Z	F	Total
1900-1910	4814	3050	3131	159	10	11164
1910-1920	7054	4634	5033	237	--	16958
1920-1930	4801	2866	3317	415	--	11399
1930-1940	3792	1523	2392	795	2	8504
1940-1950	5174	2363	3413	875	--	11825
1950-1960	9900	5672	5621	4022	6739	31954
1960-1965	8691	7986	3035	2815	33532	56059
Totals:	44226	28094	25942	9318	40283	147863

There are only 621 observations for 1964. The major bulk of data between 1953 and 1963 is from both project MAGNET and Canadian aircraft observations. Since direct measurement of total field was not done before 1953, it is certain that the few F observations listed prior to this date are computed values still erroneously indicated as measurements.

The geographic grid used to obtain data set 2 contained the same number of blocks as set No. 1, but was arranged in a different way. That is, the longitude lunes were only 1° wide and each was divided into 100 equal area segments of latitude. This geographic grid was constructed so the blocks on either side of the equator were nearly square (1° in longitude by 1.15° in latitude) and lengthened in latitude until the last area extended from 78.5° latitude to the poles.

The data selection using this second grid proceeded as before except that only data which were not selected for Set 1 were used. This initial Data Set No. 2 was noted to contain only about 40,000 observations compared to the 77,000 in Set No. 1. Although the intent of the test was to attempt to create two independent data samples, it was apparent that the survey density was sufficiently sparse over some areas and years that Data Set No. 1 had captured all of the available observations. Thus data were transferred from the first set to fill as many as possible of the grid areas of Set No. 2 for which there was not one observation per year. The total observations available now increased to 72,250. There was, then, approximately a 40% overlap of data between these two sets.

Weights

The weights w_i used were based on the estimates given in Table 1. That is $w_i = 1/\sigma^2$ for H, Z, and F and the reciprocal squares of σH and σF for D and I respectively. Although the OGO-2 data gave a somewhat lower scatter than the Vanguard 3 observations, a $\sigma = 50\gamma$ was chosen since it was not clear what biases due to orbital error or external fields might be inherent in the data.

V. Results

The computations were performed on both data sets first by finding the best set of coefficients to fit Data Set No. 1 and then the corrections to these coefficients to fit Set No. 2. The weighted rms residuals for all of the observations are as follows for Data Set No. 1:

Table 3

Component	Weighted rms (γ)	Observations
HΔD	157	44226
FΔI	199	28094
ΔZ	256	9318
ΔH	214	25942
ΔF	34	40283
Total	99	147863

The overall weighted rms residual using the output coefficients from Set 1 to the data for Set 2 was 129γ. Only one iteration was performed on Set 2 since the computation was lengthy (10 hours "Central Processor" time) and we knew by experience that the second iteration would give insignificant corrections to the coefficients. Although the magnetic tape of the second data set became unuseable before the weighted residual could be calculated using the output coefficients, we are confident that the corrections reduced the residual to about 100γ as on Set 1.

The standard errors of the Set 1 coefficients were estimated to be 1 to 2γ for the spatial terms, 0.03 to 0.08 γ/yr for the first derivatives, and .002 to .004 γ/yr^2 for the second derivatives. However, perhaps more illuminating is Table 4, which lists the coefficients of the fit to Data Set No. 1 and the correction applied to each coefficient to obtain a fit to Data Set No. 2. The changes in coefficients between the sets are very commensurate to the standard error estimates mentioned above. The question as to the absolute accuracy of the coefficients cannot be determined from this exercise since these results merely indicate that using the given set of data, weights, and number of coefficients, the smallest rms deviation is found with these parameters. The implication of the error estimates is that the coefficients are certainly not <u>more</u> precise than these values as true descriptions of the given total data set.

However, it may inspire more confidence in the field model to investigate the degree to which the total data of various classes agree with the fits. A comparison was first made by computing the unweighted residuals to a random 10% sample of <u>all</u> survey observations (including all satellite data except OGO-2) and examining the distribution of these differences. A best Gaussian curve [$D = A \exp(\Delta/\sigma)^2$] was fit to this distribution after discarding deviations whose absolute value exceeded 400γ. The sigma of this curve was 122 $\pm$ 4γ. The percentage of data falling outside a given interval compared to that predicted by the gaussian distribution is given in Table 5.

Table 4

Coefficients of Fit to Data Set No. 1
and Δ Changes to These Coefficients to Fit Set No. 2

n	m	g	Δg	h	Δh	$\dot{g}$	$\Delta\dot{g}$	$\dot{h}$	$\Delta\dot{h}$	$\ddot{g}$	$\Delta\ddot{g}$	$\ddot{h}$	$\Delta\ddot{h}$
			(γ)				(γ/yr)				(γ/yr^2)		
1	0	-30401.2	-1.6			14.03	0.07			-0.062	0.003		
1	1	-2163.8	-0.4	5778.2	-4.9	8.76	-0.23	-3.71	-0.52	0.114	-0.004	-0.043	-0.007
2	0	-1540.1	2.4			-23.29	-0.41			-0.154	-0.010		
2	1	2997.9	-1.0	-1932.0	-0.6	-0.09	-0.10	-14.31	-0.12	-0.018	-0.000	0.054	-0.004
2	2	1590.3	-1.2	202.9	0.5	-4.56	-0.32	-16.62	0.14	-0.253	-0.006	-0.016	0.003
3	0	1307.1	0.9			-0.93	0.27			-0.123	0.006		
3	1	-1988.9	0.3	-425.4	1.1	-10.62	0.10	5.20	-0.09	-0.027	-0.001	0.095	0.
3	2	1276.8	-1.1	227.8	0.1	2.31	0.17	2.53	0.02	0.028	0.004	-0.007	-0.000
3	3	881.2	-0.7	-133.8	0.3	-5.89	0.16	-6.98	-0.22	-0.183	0.003	0.079	-0.004
4	0	949.3	0.3			1.45	0.07			0.001	0.001		
4	1	803.5	-0.4	160.3	-1.7	0.90	0.01	-2.19	-0.04	-0.044	0.002	0.004	-0.003
4	2	502.9	-1.5	-274.3	-0.8	-1.75	0.07	-0.14	0.17	0.017	0.004	0.056	0.004
4	3	-397.7	0.6	2.3	-1.4	0.66	-0.26	1.88	0.10	0.007	-0.005	-0.035	0.003
4	4	266.5	1.8	-246.6	-1.8	-3.01	0.14	-6.52	-0.11	-0.097	0.002	-0.047	-0.003
5	0	-233.5	-0.6			1.61	-0.36			0.045	-0.007		
5	1	355.7	-0.5	5.1	0.4	0.60	-0.06	2.24	-0.31	0.001	-0.001	-0.046	-0.005
5	2	228.4	-0.9	117.8	-0.3	3.34	-0.08	1.59	0.06	0.075	-0.001	0.007	0.002
5	3	-28.8	0.2	-114.8	-0.2	-0.04	0.13	-2.61	0.11	0.008	0.002	-0.007	0.004
5	4	-157.9	-0.4	-108.9	0.0	-0.60	0.14	0.50	0.00	0.015	0.004	0.001	0.
5	5	-62.2	0.7	82.4	-0.7	1.76	-0.17	-0.12	0.01	0.056	-0.003	-0.024	0.000
6	0	49.2	-0.5			-0.42	0.06			-0.006	0.002		
6	1	57.5	0.2	-12.1	-2.5	0.82	-0.02	0.05	0.15	0.015	-0.002	0.020	0.001
6	2	-0.8	-1.2	104.4	0.3	0.82	-0.14	0.09	-0.02	0.010	-0.003	-0.011	-0.001
6	3	-238.3	-0.1	56.6	1.7	2.35	0.05	2.55	-0.05	0.050	0.001	0.015	-0.002
6	4	-1.5	1.5	-23.4	-0.7	0.83	-0.15	-1.19	0.12	-0.011	-0.005	-0.029	0.004
6	5	-2.0	0.5	-14.8	0.4	0.01	0.05	0.33	0.01	0.026	0.001	0.029	0.001
6	6	-108.9	0.4	-13.3	0.6	0.23	-0.02	0.84	0.15	0.023	0.000	-0.010	0.001
7	0	72.2	0.4			-0.57	-0.61			-0.014	-0.012		
7	1	-53.7	-0.7	-53.7	2.1	-0.34	-0.12	-0.96	-0.13	-0.006	-0.002	-0.014	-0.001
7	2	7.9	0.6	-27.4	-1.0	-1.44	-0.42	0.01	0.13	-0.034	-0.010	0.016	0.004
7	3	15.6	-0.9	-8.1	-0.4	-0.90	0.03	0.43	0.36	-0.004	0.002	0.014	0.008
7	4	-24.3	0.8	7.0	0.1	0.03	-0.06	0.75	-0.13	-0.006	-0.000	0.005	-0.004
7	5	-3.6	-0.8	24.3	0.8	-0.60	-0.02	-0.33	-0.22	-0.027	-0.000	-0.008	-0.005
7	6	15.5	-0.3	-22.5	0.6	-0.17	0.06	0.49	-0.03	-0.001	0.001	0.016	-0.001
7	7	3.6	0.4	-21.4	-0.4	-0.64	0.20	0.90	-0.09	-0.004	0.004	0.011	-0.001
8	0	8.5	-2.7			0.35	0.39			0.006	0.009		
8	1	6.5	-0.8	5.4	-1.3	0.50	0.24	-0.50	0.43	0.008	0.004	-0.015	0.009
8	2	-9.3	-2.1	-11.7	-0.1	1.70	0.48	-0.21	-0.03	0.039	0.011	-0.012	-0.002
8	3	-9.6	-1.2	4.2	1.0	-0.11	0.15	0.03	-0.02	-0.008	0.003	0.005	-0.001
8	4	-6.1	-0.1	-15.3	0.6	0.34	0.21	-0.79	-0.11	0.015	0.003	-0.011	-0.002
8	5	5.5	-0.7	4.6	-0.3	-0.07	0.11	0.05	0.03	-0.002	0.003	-0.000	0.000
8	6	-8.1	-0.4	21.9	0.0	0.43	-0.06	0.10	-0.12	0.005	-0.001	-0.003	-0.003
8	7	13.0	0.5	-0.7	0.6	-0.15	0.02	-0.36	-0.04	-0.008	0.001	-0.009	-0.001
8	8	7.4	-0.5	-17.1	0.4	-0.42	0.02	-0.43	-0.06	-0.007	0.001	-0.003	-0.002
9	0	10.4	0.0			-0.10	-0.56			-0.005	-0.011		
9	1	5.8	-0.0	-22.4	3.0	-0.13	-0.46	0.66	-0.28	-0.001	-0.010	0.022	-0.008
9	2	7.5	1.8	13.8	1.2	-1.20	-0.53	0.54	0.26	-0.027	-0.011	0.007	0.006
9	3	-15.1	1.4	6.3	1.1	0.08	-0.21	0.03	0.10	0.005	-0.006	-0.002	0.000
9	4	12.1	1.8	-3.0	0.0	-0.08	-0.10	0.35	0.05	-0.007	-0.002	0.009	0.000
9	5	4.7	0.9	-1.9	-0.6	-0.39	-0.22	-0.03	0.12	-0.006	-0.006	0.006	0.003
9	6	0.2	0.7	9.0	0.9	-0.36	-0.15	-0.01	-0.16	-0.009	-0.003	-0.001	-0.004
9	7	1.6	-0.8	11.5	-1.4	0.47	0.11	0.45	0.18	0.006	0.002	0.009	0.005
9	8	0.9	0.4	0.1	0.1	0.37	0.12	-0.05	0.05	0.005	0.002	-0.004	0.001
9	9	0.2	1.3	-1.5	0.9	-0.46	-0.01	0.75	0.26	-0.009	-0.001	0.019	0.004
10	0	-2.9	-2.8			-0.01	-0.26			-0.003	-0.005		
10	1	-0.9	-0.8	-0.1	2.4	-0.13	0.26	-0.61	0.88	-0.003	0.005	-0.012	0.019
10	2	-2.2	-0.9	4.5	-0.7	0.88	0.21	-0.64	0.05	0.020	0.004	-0.014	0.001
10	3	0.8	-1.0	-1.0	2.3	-0.18	0.37	0.02	0.30	-0.008	0.009	0.001	0.006
10	4	-2.8	0.4	2.6	-1.0	0.17	0.10	0.05	0.05	0.007	0.002	0.001	0.002
10	5	6.4	-0.5	-4.4	0.8	-0.02	0.20	-0.63	-0.27	0.001	0.004	-0.011	-0.006
10	6	4.7	-1.0	-1.3	-1.3	0.05	0.14	-0.07	0.13	0.001	0.004	-0.001	0.003
10	7	-0.2	-0.1	-3.6	0.8	0.17	-0.01	0.07	0.01	0.001	0.000	0.001	-0.001
10	8	1.8	0.5	4.0	0.0	0.16	-0.14	-0.03	-0.02	0.005	-0.004	-0.001	-0.000
10	9	2.0	-0.0	1.0	0.4	0.31	0.09	-0.02	-0.00	0.004	0.002	0.001	0.000
10	10	1.1	1.1	-2.0	-0.3	-0.23	0.14	-0.45	0.09	-0.002	0.002	-0.006	0.002

Table 5

$\|\Delta\|$	Gaussian Prediction (σ = 122γ)	Data
100γ	41%	51%
200	10	25
300	1.4	13
400	0.1	8
500	0.0	5

The Δ value here as before is either an observed force component (ΔF, ΔH, ΔZ) or an angle converted to a force (i.e. $H\Delta D$, $F\Delta I$). The data distribution contains a considerable 'tail' beyond an almost gaussian center. Of course, since no smoothing was done to the data, magnetic anomalies are present and are the main reason for the non-gaussian distribution. A breakdown of the data into classes was also done and the gaussian constant fit to these distributions is given in Table 6.

Table 6

Gaussian Sigmas of Curves Fit to Distribution
Of Residuals From GSFC(12/66)-1 Field

	Component					
Type	$H\Delta D$	$F\Delta I$	ΔH	ΔZ	ΔF	All
Surface	120	150	120	130	120	125
Aircraft	170	160	180	210	100	140
Vanguard 3-Alouette					28	28
All	130	150	130	170	100	122

The aircraft F observations (all project MAGNET) are seen to be below average in spite of the fact that the component aircraft differences are greater than those for the surface measurements. The Z observations are above average for all data. The combined Vanguard 3-Alouette figure is mainly a reflection of the Vanguard 3 data due to the much larger ($\sim$ 3000) number of observations. The Alouette data consist only of a few hundred observations which scatter at about 100γ.

A separate distribution was calculated for the OGO-2 data. Its gaussian constant is only 11.7γ; a value much narrower than for the other data as might be expected since the observations are more accurate, are taken above an altitude where crustal anomalies are effective, and are numerous and heavily weighted. Table 7 gives the percentage data which have residuals outside of given limits as compared with that of a gaussian distribution of $\sigma = 11.7\gamma$. As can be seen here, there is the slight skewing on the low field side [ΔF = F(measured) - F(computed)] as noted before for Vanguard 3 data (Cain et al., 1962). This asymmetry is likely due to occasional weak magnetic disturbance during the fairly quiet interval from which the data were selected. The residual distribution on the high field side is remarkably gaussian.

Table 7

% Data Outside Given ΔF Limit

Field Low Data	Field Low ΔF	Gaussian Prediction ($\sigma = 11.7\gamma$)	Field High ΔF	Field High Data
24%	-10γ	21%	10γ	18%
10	-20	5	20	5
4	-30	0.6	30	1
1	-40	.03	40	.5

RMS residuals were also computed for the selected data set 1 for each year 1900-1964 to see whether there were any uniform deviations which could be attributed to the inadequacy of a parabolic function in time for each coefficient to represent secular change and if there was any obvious correlation with magnetic disturbance. A suggestion was made previously (Cain, 1966, p. 22) that a maximum noted in the residuals between the GSFC(7/65) field and survey data for the years 1957-1958 might be attributed to the larger incidence of magnetic disturbance near sunspot maxima. If this effect were real, one should also note some systematic enhancement of the residuals from the present fit near the sunspot maxima years 1906, 1917, 1928, 1937, 1947, and 1957. A plot of the rms residuals for all components by year is given in the top curve of Figure 2. Although the 1958 peak is still present there is no obvious correlation between the other peaks in this

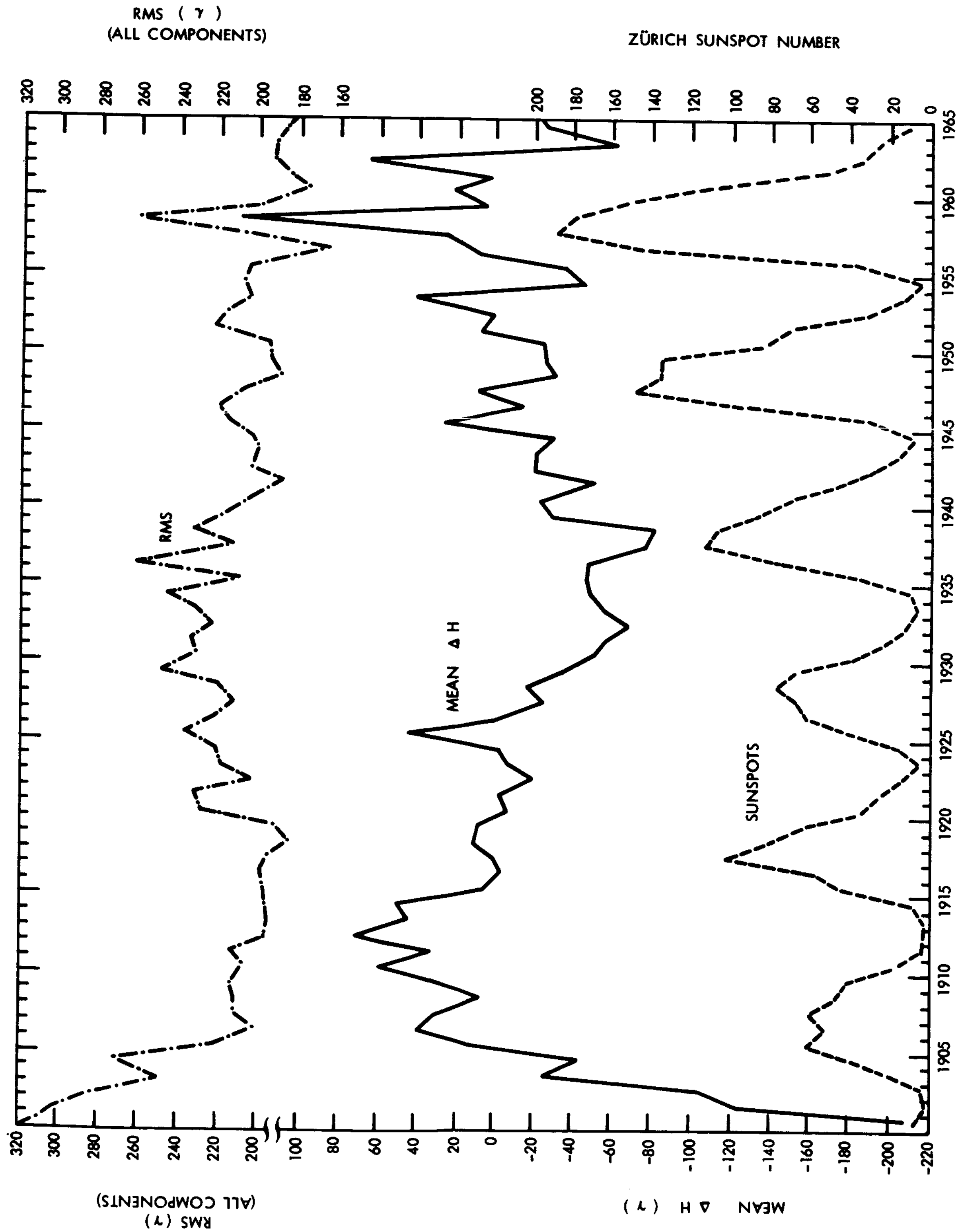
RMS (γ)
(ALL COMPONENTS)
ZÜRICH SUNSPOT NUMBER
RMS
MEAN ΔH
SUNSPOTS
MEAN ΔH (γ)
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965

Figure 2

curve and the sunspot cycle curve shown at the bottom. Chernosky (1966) has shown that the magnetic activity-sunspot correlation may not be very simple and thus a fuller investigation may be more revealing.

We have also calculated the mean differences for the H observations by year and show a plot of these results in the middle curve of Figure 2. Again there appears to be no obvious correlation with the sunspot cycle. There does, however, appear to be a systematic lowering of the curve for the years 1900-1906 and 1929-1944 as compared with other years. The general shape of the two curves of these residuals indicates that there are probably significant systematic deviations yet existing in the data which indicate that the secular change coefficients determined may not be the best estimate over the whole interval. Further study is thus indicated to determine whether the sharp peaks in rms curves are real or are due only to a few erroneous or anomalous data. A detailed comparison also needs to be made with the secular change as predicted by this numerical fit and that measured at the magnetic observatories.

Reality of Computed Field

Having compared the way in which the GSFC(12/66)-1 field matches the existing data, one is still faced with the question of how well it represents the actual geomagnetic field for current epochs and how well it extrapolates into the near future. As we have noted before (Cain, 1966, p. 19) any standard deviations which can be computed from such fits are smallest near the regions occupied by data and increase away from such regions. Since the fields computed from the two data sets are presumably of equivalent quality, one test of some value is to tabulate the differences in actual field that arise between them. A summary of the maximum absolute differences (1900-1970) for each component on the earth's surface and the location where this occurs is given in Table 8. As can be seen in this table, the location of maximum differences is, without exception, in or near Antarctica for any of the force components and near the magnetic poles for declination. This search was made using only the 10^{o} latitude-longitude grid intersections so that the changes from year to year are by that increment. The maximum differences for the inclination occur near the magnetic equator in the area of the Pacific which has traditionally been sparsely surveyed. The locations of the area of maximum differences are not surprising if one notes the data distribution by area and epoch indicated in Figure 1. The lack of substantial polar data, particularly over and near Antarctica for any past epoch, means that the potential expansion is relatively free to assume imaginary values in these regions. The absolute differences between the fields computed from the two coefficient

Table 8

Maximum Absolute Differences between Field Components Computed by GSFC(12/66)-1 and GSFC(12/66)-2 and locations of differences. Zero altitude.

	X		Y		Z		D		I		F	
	γ		γ		γ		°		°		γ	
1900	400	50S,120W	330	70S,150W	510	60S,110W	7	70N,90W	.5	0,110W	560	60S,110W
1905	260	50S,120W	240	70S,150W	370	70S,110W	4	70N,90W	.3	30S,110W	390	60S,110W
1910	140	50S,120W	160	70S,150W	270	70S,110W	4	70N,150E	.2	30S,110W	240	70S,110W
1915	170	80S,110E	150	80S,170E	230	70S,110E	10	70S,150E	.2	10S,10W	230	70S,110E
1920	210	80S,100E	210	80S,150E	290	70S,110E	8	70S,150E	.2	10S,110W	290	70S,100E
1925	240	80S,100E	250	80S,150E	330	70S,100E	5	70S,140E	.3	10S,110W	330	70S,100E
1930	260	80S,100E	280	80S,150E	360	70S,100E	6	70S,140E	.3	0,110W	350	70S,100E
1935	260	80S,100E	280	80S,150E	370	70S,100E	6	70S,140E	.3	0,110W	360	70S,90E
1940	250	80S,100E	280	80S,140E	360	70S,100E	6	70S,140E	.3	0,110W	350	70S,90E
1945	230	80S,90E	250	80S,140E	330	70S,100E	5	70S,140E	.3	0,110W	320	70S,90E
1950	190	80S,90E	210	80S,140E	280	70S,100E	4	70S,140E	.2	0,110W	280	70S,90E
1955	140	80S,100E	140	80S,140E	220	70S,100E	2	70S,140E	.1	0,110W	210	70S,90E
1960	90	60S,110E	60	80S,150E	130	70S,100E	2	80N,110W	.1	20S,110W	130	70S,100E
1965	130	70S,90W	90	70S,60W	200	60S,90W	7	80N,110W	.2	10S,110W	200	60S,90W
1970	210	70S,90W	160	80S,140W	300	60S,90W	16	80N,110W	.4	10S,110W	300	60S,90W

sets should be representative of the minimum possible errors that can arise from this data sparsity and the improvements that are possible for recent epochs. Of course, the OGO-2 data constrains the potential function considerably at the 1965.8 epoch and hopefully contributes substantially to the realism of the current field whereas the past data aids in the reality of the secular change. Thus although the field in the south polar region may be well determined for the first time, the secular change in this area will likely be uncertain.

If we average the square of the differences of the field components computed from the two coefficient sets over the earth's surface we obtain the results in Table 9. These differences were computed at each 10^{o} intersection of latitude and longitude and were weighted by the cosine of latitude. The fits are seen to be in best agreement during the period of maximum data concentration (1955-1965) and diverge by a factor of about two as an attempt is made to extrapolate beyond the limits of available data to 1970. We judge that the numbers in this table are comparable to the average errors in either estimate of the main core field near the earth's surface. Although a given measurement of a field component on the earth's surface may deviate from that computed due to the anomaly "noise" according to Table 5 (with some variation between components as noted in Table 6), we would be surprised if the values in Table 9 were less than half of the true standard error of the field model.

The rms difference between the two fields becomes smaller with increasing altitude. A computation similar to that for Table 9 except

Table 9

RMS Differences between GSFC(12/66)-1 and (12/66)-2 at zero altitude

	Xγ	Yγ	Zγ	D°	I°	Fγ
1900	85	56	109	.36	.12	104
1905	57	41	76	.22	.09	72
1910	38	31	53	.19	.06	50
1915	30	27	44	.33	.04	42
1920	34	28	48	.33	.04	46
1925	42	32	56	.29	.05	54
1930	48	34	63	.31	.06	61
1935	51	35	67	.31	.06	64
1940	50	35	66	.29	.06	63
1945	46	32	60	.25	.06	57
1950	38	26	50	.19	.05	48
1955	27	20	36	.13	.04	34
1960	18	14	25	.13	.03	22
1965	27	17	34	.26	.04	30
1970	49	30	62	.48	.07	57

for an altitude of 1000 Km gives residuals that are smaller by a factor of 3 to 4 for the force components X, Y, Z, F, and by a factor of two for the angular components. Although the fit was itself more rigidly constrained by the high weighting factor and narrow scatter of the OGO-2 data so that is might be predicted that the two fits would agree well in the altitude range of these data, these smaller differences may also be explained by the fact that the relative effect of the uncertainties in the higher order coefficients lessens with increasing altitude. That is, in the computation of the field from the potential, the expansion for each component contains a term $(a/r)^{n+2}$. Thus the contribution of each coefficient of degree n=10 is decreased by a factor of about 6 at 1000 km. altitude over that at the earth's surface whereas the same factor for the dipole terms only decreases their effect by about 35%.

VI. Conclusions

We conclude from this study that the field derived from the GSFC(12/66)-1 set of coefficients is an extremely good model for recent epochs and because of the inclusion of the very comprehensive sample of OGO-2 satellite data should be the most accurate measure of the main field currently available. The maximum errors expected at the earth's surface still lie in Antarctic regions and are expected to be up to 200γ in F and a few tenths of a degree in dip. Due to the thinness of observations over a long period in southern areas extrapolation into the future remains more hazardous than elsewhere. In other areas the surface field is probably no further in error than a few tens of gammas and the growth of error by extrapolation into the future is only of the order of a factor of two by 1970.

Future improvements in this representation are indicated when a greater sophistication is introduced by taking into account time variations in the satellite measurements and when data become available for more recent epochs. Some slight changes will probably be necessary when the more definitive orbits of OGO-2 are available. There is the possibility that the systematic errors in the orbits and the corrections due to time variations could bring about distortions of the order of the 10-20γ errors estimated at satellite altitudes.

However, for most uses of an IGRF such additional accuracy is not of prime concern, and the present results could be taken as more than adequate as an interim field for 1965.0 prior to the final evaluations to be considered for the results of the World Magnetic Survey.

APPENDIX

Weighted Least Square Fitting to Main Field Data

This formulation is an expansion of that given by Cain et al., (1965) including the correction of a few errors. The recent review by Kaula (1967) is also valuable to consult.

Given the functional form $C(p; r, \theta, \phi, t)$ as representing any measure of the geomagnetic field we attempt to minimize the expression

$$\chi^2 = \sum_i \left[C_i - C(p; r, \theta, \phi, t)\right]^2 w_i$$

where p is here taken to be any of the n parameters of the field, (r, θ, ϕ, t) are the coordinates of the observation C_i, and w_i is a chosen weighting factor. The summation is over some chosen set of data.

Expanding C in a Taylor's series about some approximate solution, C_o gives:

$$\chi^2 = \sum_i \left[C_i - C_0 - \sum_{k=1}^{n} \left(\frac{\partial C}{\partial p_k}\right)_0 \delta p_k - \frac{1}{2!} \sum_{j,k=1}^{n} \left(\frac{\partial^2 C}{\partial p_j\, \partial p_k}\right)_0 \delta p_j\, \delta p_k \cdots \right]^2 w_i$$

so that taking the derivative of χ^2 with respect to the <u>corrections</u> to the parameters, solving for the k^{th} normal equation, and neglecting all but the linear terms gives:

$$\sum_{j=1}^{n} \delta p_j \sum_i w_i \left(\frac{\partial C}{\partial p_j}\right)_0 \left(\frac{\partial C}{\partial p_k}\right)_0 = \sum_i w_i (C_i - C_0) \left(\frac{\partial C}{\partial p_k}\right)_0$$

We now let
$$D_{jk} = \sum_i w_i \left(\frac{\partial C}{\partial p_j}\right)_0 \left(\frac{\partial C}{\partial p_k}\right)_0$$

and
$$W_k = \sum_i w_i \left(C_i - C_0\right) \left(\frac{\partial C}{\partial p_k}\right)_0$$

where the subscript o indicates that the derivatives are to be evaluated at the values $p = p_o$ for the (r, θ, ϕ, t) of an i^{th} observation. The solution to these equations thus becomes:

$$\delta p_j = \sum_{k=1}^{n} D_{jk}^{-1} W_k$$

where D^{-1} are the elements of the matrix inverse of D.

In order to develop the detailed expressions from the measured components that enter the above solution we first write the potential of the field as

$$V = a \sum_{n=1}^{n\,max} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) P_n^m(\theta)$$

where r, θ, ϕ are the geocentric coordinates, P_n^m the Schmidt normalized spherical functions and a (= 6371.2 km.) a scale factor chosen arbitrarily to be the earth's mean radius. Taking $\bar{F} = -\nabla V$, the components are thus

$$F_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{n\,max} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) \frac{\partial P_n^m(\theta)}{\partial \theta}$$

$$F_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} = \frac{1}{\sin\theta}\sum_{n=1}^{nmax}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m\left(g_n^m \sin m\phi - h_n^m \cos m\phi\right) P_n^m(\theta)$$

$$F_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^{nmax}\left(\frac{a}{r}\right)^{n+2}(n+1)\sum_{m=0}^{n}\left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) P_n^m(\theta)$$

For the calculation of the P_n^m and $\frac{\partial P_n^m}{\partial\theta}$ we go through a two-step process in which the Gauss-Laplace functions $P^{n,m}$ and $\frac{\partial P^{n,m}}{\partial\theta}$ are first evaluated from the relations:

$$P^{0,0} = 1 \qquad \frac{\partial P^{0,0}}{\partial\theta} = 0$$

$$P^{n,n} = (\sin\theta)\,P^{n-1,n-1} \qquad \frac{\partial P^{n,n}}{\partial\theta} = (\sin\theta)\frac{\partial P^{n-1,n-1}}{\partial\theta} + (\cos\theta)\,P^{n-1,n-1}$$

where $n \geq 1$; and, for $m \neq n > 1$,

$$P^{n,m} = (\cos\theta)\,P^{n-1,m} - K^{n,m}P^{n-2,m}$$

$$\frac{\partial P^{n,m}}{\partial\theta} = (\cos\theta)\frac{\partial P^{n-1,m}}{\partial\theta} - (\sin\theta)\,P^{n-1,m} - K^{n,m}\frac{\partial P^{n-2,m}}{\partial\theta}$$

where
$$K^{n,m} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)}$$

(Note that the evaluation of the $n = 1$, $m = 0$ terms constitutes no problem using these relations in spite of the fact that $P^{-1,0}$ and $\frac{\partial P^{-1,0}}{\partial\theta}$ are undefined. In this case the $K^{1,0}$ multiplier is zero.)

The Schmidt functions are then defined by multiplying by the factors $S^{n,m}$ where

$$S^{0,0} = 1 \qquad S^{n,0} = S^{n-1,0}\left[\frac{2n-1}{n}\right]$$

and $$S^{n,m} = S^{n,m-1}\sqrt{\frac{(n-m+1)J}{n+m}} \qquad \begin{array}{l}\text{where } J = 2 \text{ for } m = 1\\ \phantom{\text{where }} J = 1 \text{ for } m > 1\end{array}$$

In the evaluation of the field components from the derived g_n^m, h_n^m it is generally more efficient to multiply the coefficients by the above factors once rather than to perform the additional $n(n + 3)$ multiplications for the evaluation at each θ. (Cain et al., 1965). The resulting coefficients $g^{n,m}$ and $h^{n,m}$ are also different by convention from the g_n^m, h_n^m by a reversal of sign. If these "Gauss normalized" coefficients are used directly the field components should be caluclated from $+\nabla V$ instead of $-\nabla V$ as given above.

For example

$$P_1^0 = P^{1,0} = \cos\theta$$

$$\frac{\partial P_1^0}{\partial\theta} = \frac{\partial P^{1,0}}{\partial\theta} = -\sin\theta$$

Thus for the axial dipole at $r = a$

$$F_\theta = -g_1^0(-\sin\theta) = g_1^0\sin\theta = -g^{1,0}\sin\theta$$

Since for the earth's field $g_1^o < 0$, then $F_\theta < 0$ for all $0 < \theta < \pi$.

With the exception of the satellite data, all of the observed components C_i are given in a geodetic coordinate system. For the computation of the corresponding values of C_o and their derivatives relative to the coefficients, we first convert from the geodetic coordinates λ(latitude) and h(altitude above geoid) to geocentric radius r and the colatitude θ functions, $\sin\theta$ and $\cos\theta$, as follows:

$$\cos\theta = \frac{\sin\lambda}{\sqrt{t'\cos^2\lambda + \sin^2\lambda}}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$r = \left[h\left(h + 2\sqrt{a^2 - \left(a^2 - b^2\right)\sin^2\lambda}\right) + \frac{a^4 - \left(a^4 - b^4\right)\sin^2\lambda}{a^2 - \left(a^2 - b^2\right)\sin^2\lambda}\right]^{1/2}$$

where

$$t' = \left[\frac{h\sqrt{a^2 - \left(a^2 - b^2\right)\sin^2\lambda} + a^2}{h\sqrt{a^2 - \left(a^2 - b^2\right)\sin^2\lambda} + b^2}\right]^2$$

and the factors involving the earth's equatorial radius a and polar radius b are calculated from the value a (= 6378.165 km. and flattening, f(= (a - b)/a = 1/298.25) from the relations

$$f_1 = 1 - f$$

$$b^2 = (af_1)^2$$

$$(a^2 - b^2) = a^2(1 - f_1^2)$$

$$(a^4 - b^4) = a^4(1 - f_1^4)$$

The computed values of the ordinarily measured geodetic components of field X(north), Y(east), and Z(vertical) are then given from the $\bar{F}$ calculated at these geocentric positions by the rotation

$$\bar{X} = \bar{F}\,\bar{R}(\delta) \quad \text{or}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -\cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} F_\theta \\ F_\phi \\ F_r \end{pmatrix}$$

where $\delta = \lambda + \theta - \pi/2 \geq 0$ and the trigonometric functions used are computed from

$$\sin\delta = \sin\lambda \sin\theta - \cos\lambda \cos\theta$$

$$\cos\delta = \sqrt{1 - \sin^2\delta}$$

The values of the other components are then

$$H = \sqrt{X^2 + Y^2}$$

$$F = \sqrt{H^2 + Z^2}$$

$$D = 2 \tan^{-1}[Y/(X + H)]$$

$$I = 2 \tan^{-1}[Z/(F + H)]$$

where $-\frac{\pi}{2} < \tan^{-1} < \frac{\pi}{2}$

The derivatives of the measured orthogonal components X,Y,Z relative to the coefficients g_i and h_i in the expansions

$$g(t) = g_0 + g_1 \Delta t + g_2 \Delta t^2 = g_i (\Delta t)^i$$

$$h(t) = h_0 + h_1 \Delta t + h_2 \Delta t^2 = h_i (\Delta t)^i$$

can be represented by the derivatives of $\overline{F}$ multiplied by the rotation $\overline{R}(\delta)$ and $(\Delta t)^i$. That is:

$$\begin{pmatrix} \frac{\partial X}{\partial g_i} & \frac{\partial X}{\partial h_i} \\ \frac{\partial Y}{\partial g_i} & \frac{\partial Y}{\partial h_i} \\ \frac{\partial Z}{\partial g_i} & \frac{\partial Z}{\partial h_i} \end{pmatrix} = (\Delta t)^i \begin{pmatrix} -\cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} \frac{\partial F_\theta}{\partial g_i} & \frac{\partial F_\theta}{\partial h_i} \\ \frac{\partial F_\phi}{\partial g_i} & \frac{\partial F_\phi}{\partial h_i} \\ \frac{\partial F_r}{\partial g_i} & \frac{\partial F_r}{\partial h_i} \end{pmatrix}$$

Considering the equations for $\overline{F}$, the last matrix is seen to be

$$\begin{pmatrix} \frac{\partial F_\theta}{\partial g_i} & \frac{\partial F_\theta}{\partial h_i} \\ \frac{\partial F_\phi}{\partial g_i} & \frac{\partial F_\phi}{\partial h_i} \\ \frac{\partial F_r}{\partial g_i} & \frac{\partial F_r}{\partial h_i} \end{pmatrix} = \left(\frac{a}{r}\right)^{n+2} \begin{pmatrix} -\frac{\partial P}{\partial \theta} \cos m\phi & -\frac{\partial P}{\partial \theta} \sin m\phi \\ \frac{m}{\sin\theta} P \sin m\phi & -\frac{m}{\sin\theta} P \cos m\phi \\ (n+1) P \cos m\phi & (n+1) P \sin m\phi \end{pmatrix}$$

so that each of the derivatives contains the common terms $(\Delta t)^i \left(\frac{a}{r}\right)^{n+2}$.
The remaining factors thus become as follows:

Component	Parameter	Factors
X	g	$\cos m\phi \left[\frac{\partial P}{\partial \theta} \cos\delta - (n+1) P \sin\delta\right]$
X	h	$\sin m\phi \left[\frac{\partial P}{\partial \theta} \cos\delta - (n+1) P \sin\delta\right]$
Y	g	$(m/\sin\theta) \sin m\phi$
Y	h	$-(m/\sin\theta) \cos m\phi$
Z	g	$-\cos m\phi \left[\frac{\partial P}{\partial \theta} \sin\delta + (n+1)P \cos\delta\right]$
Z	h	$-\sin m\phi \left[\frac{\partial P}{\partial \theta} \sin\delta + (n+1)P \cos\delta\right]$

Thus the partials $(\frac{\partial C}{\partial p})$ may be derived from the above factors by multiplying by $(\Delta t)^i \left(\frac{a}{r}\right)^{n+2}$.

The partials $(\frac{\partial C}{\partial p})$ for the other measured components of the geomagnetic field such as D, I, H, and F are derived from the previously derived partials using the following relations:

Since $\quad D = \tan^{-1}(Y/X)$

then $\quad dD = \frac{\partial D}{\partial Y} dY + \frac{\partial D}{\partial X} dX = \frac{X}{H} dY - \frac{Y}{H} dX$

from which $\quad \frac{\partial D}{\partial g} = \frac{X}{H} \frac{\partial Y}{\partial g} - \frac{Y}{H} \frac{\partial X}{\partial g}$

and $\quad \frac{\partial D}{\partial h} = \frac{X}{H} \frac{\partial Y}{\partial h} - \frac{Y}{H} \frac{\partial Y}{\partial h}$

Likewise, using the relations

$$I = \tan^{-1}(Z/H)$$

$$H = \sqrt{X^2 + Y^2}$$

and $$F = \sqrt{X^2 + Y^2 + Z^2}$$

we obtain the expressions

$$dI = \frac{H}{F} dZ - \frac{XZ}{HF} dX - \frac{YZ}{HF} dY$$

$$dH = \frac{X}{H} dX + \frac{Y}{H} dY$$

$$dF = \frac{X}{F} dX + \frac{Y}{F} dY + \frac{Z}{F} dZ$$

from which the appropriate partials are easily obtained.

In conclusion one should note that this formulation does not include expressions for external terms. Also, although it is developed to fit data taken in geodetic coordinates, a simpler derivation is possible if the data could be rotated into a geocentric system. This rotation was not attempted here since there is such a large fraction of the total data for which only one component is given.

The translation of these relations into an operating computer program is not difficult but it should be noted that if corrections are made on all of the coefficients simultaneously the number of computer core locations required is rather large. For example, for a maximum value of n and m of 10 the size of the triangular array for D_{jk} needed to correct on all six g_i, h_i is of the order of 66000. Since D is inverted to solve for the corrections it is necessary to use floating point word fractions in excess of 27 binary bits for a successful inversion with 48 bits an adequate size to maintain accuracies of $\pm$ 1γ.

The main computation time for the corrections is spent on forming the original matrix values D_{jk} and W_k for each observation. The computation of the partials $(\frac{\partial C}{\partial p_i})$ for each observation and the inversion of D_{jk} require only a small fraction of the total time.

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